Exam Nonlinear Optics

Wednesday, April 23, 2008, 9.00-12.00 (room 5113.0201)

Give your name on each sheet.
On the first sheet, also give your student number
and the total number of sheets turned in.
Success!

Problem 1 (6 points)
Consider a material consisting of one-dimensional classical anharmonic oscillators with a quadratic force term $-m a x^2$ as nonlinearity. The density of oscillators is denoted N. Assume that a c.w. light beam is incident: $\tilde{E} = E\exp[-i\omega t] + c.c.$

Derive the susceptibilities: $\chi^{(1)}(\omega)$, $\chi^{(2)}(2\omega)$, and $\chi^{(3)}(3\omega)$. Explain the quantities you introduce.

Problem 2 (5 points)
Consider a sum-frequency generation process.

a) If both fundamental (incoming) pulses are assumed undepleted, how does the signal intensity then depend on the length L of the sample? (Calculation is allowed, but not necessary.)

b) An important factor in this dependence is the wavevector mismatch. How is this quantity defined? Explain physically why it determines the signal intensity?

c) Now assume that one of the fundamental pulses is depleted, while the other is not. Draw the intensity of all pulses as a function of propagation distance in the sample for zero wavevector mismatch. How would your drawing change if we increase $\chi^{(2)}$ by a factor of 2?

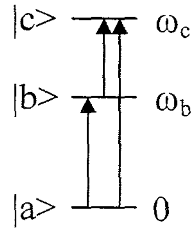
Problem 3 (8 points)
Consider the molecular energy level diagram below, with three levels of interest. The dephasing rate of the coherence $|a\rangle\langle b|$ is given by γ_{ab}, and analogous for the other coherences. None of the levels has a permanent dipole.

a) Show by general consideration of the structure of Feynman diagrams that all three transitions indicated by arrows should be dipole-allowed (i.e., have a nonzero dipole matrix element) in order for this system to have a second-order optical response.

b) We are interested in sum-frequency generation, with incoming fields of frequencies ω_1 and ω_2. Let us assume that $\omega_1 \approx \omega_b$ and $\omega_2 \approx \omega_{cb}$. Show that at least one Feynman diagram exists that gives a double resonance; draw this diagram.

c) Calculate the diagram which you have found in b). In case you did not find a diagram with the desired property, give an arbitrary other diagram which contributes to SFG in this system and calculate it.

d) Is the assumed arrangement of nonzero transition dipoles allowed for a centrosymmetric system? Motivate your answer.



Problem 4 (7 points)
Consider an ensemble of two-level molecules in a host medium. As a consequence of inhomogeneity in the host, each molecule has a slightly different transition frequency. These frequencies are distributed according to a Gaussian distribution with mean ω_0 and standard deviation D. The coherence between the ground state and the excited state of each molecule decays exponentially with the same homogeneous dephasing rate γ.

a) Give an expression for the absorption spectrum of the ensemble in the limit $D \gg \gamma$. Same question if $D \ll \gamma$. (Note: No derivation is needed. You do not need to give all prefactors; the frequency dependence is the aspect of interest.)

b) What is free-induction decay (also known as inhomogeneous dephasing)? What is the physical explanation? What is the timescale for free-induction decay in the above example? (No derivation needed.)

c) Explain how one can use the two-pulse echo experiment to measure the homogeneous dephasing rate, even if $D \gg \gamma$. You do not need to give a complete calculation; a clear analysis of phase factors and relaxation factors suffices.